Semi-Annual Status Report

submitted to

NASA
Goddard Space Flight Center
Greenbelt, Maryland

Error Control Techniques for Satellite and Space Communications

Grant Number NAG5-557
June 1, 1985 - May 31, 1986

Principal Investigator:

Daniel J. Costello, Jr.
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

January, 1986

(NASA-CR-176568) ERROR CONTROL TECHNIQUES FOR SATELLITE AND SPACE COMMUNICATIONS Semiannual Status Report, 1 Jun. 1985 - 31 May 1986 (Notre Dame Univ.) 22 p HC A02/MF A01 CSCL 17B G3/32

N86-16453

Unclas
05150

Summary of Progress

During the first six months of the current grant period, progress was made in the following areas:

1) Undetected Error Probability and Throughput Analysis of a Concatenated Coding Scheme.

A paper was submitted to the IEEE Transactions on Communications summarizing our work on the performance analysis of NASA's telecommand system. A copy of this paper is included with this report. This work was initiated under a previous grant (NAG5-234) and completed in the current grant period. Our analysis assumes that the decoded frames are scrambled (interleaved) prior to decoding by the outer code. This assumption simplifies the analysis for the outer code and yields similar results to those obtained by Lin and Kasami under NASA grant NAG5-407.

Because of the interleaving assumption, it is not necessary to know the detailed weight structure of the erroneously decoded frames but only the average decoded bit error rate. This allows us the possibility of evaluating many different frame coding options. One of the most interesting was to consider a (63,56) frame code decoded using a soft decision Viterbi decoder with a $2^7 = 128$ state trellis. This has two advantages over the standard method of decoding this code (as a distance -4 Hamming code). First, soft decisions yield improved decoder performance over hard decision decoding. Second, a variable threshold can be set on the decoding trellis metrics which allow one to exchange system reliability for throughput. A tightened metric threshold allows only the most reliable frames to be decoded, resulting in very high reliability at a lower throughput. A looser metric threshold will improve throughput with some sacrifice in reliability. It is recommended that further study of this

frame coding option be undertaken to compare its performance in detail to that of the standard hard decision decoding option.

2) Capacity and Cutoff Rate Analysis of Concatenated Codes.

A paper was submitted to the IEEE Transactions on Information Theory summarizing our work on analyzing the capacity and cutoff rate of the "outer channel" formed by the combination of the actual physical channel and the inner encoder and decoder in a concatenated coding system. A copy of this paper is included with this report. The purpose of this study is to determine the best combination of inner and outer codes to use in a concatenated coding system. The submitted paper covers the case where the inner code is a block code. This work, begun under a previous grant (NAG5-234), has established that in general (1) it is better not to interleave between the inner and outer codes, and (2) for a fixed overall code rate, it is better to use higher rate inner codes and lower rate outer codes. This suggests that systems such as NASA's TDRS system could achieve better performance with a higher rate (greater than 1/2) inner code and a lower rate outer code (with more than 32 check symbols). It also suggests that if higher rate, more bandwidth efficient codes are needed, the rate of the inner code, rather than the outer code, should be increased. Work is continuing on this problem with the case of inner convolutional codes being considered. The analysis is more difficult in this case because the inner decoder error events do not appear in blocks of fixed length but can be of many different lengths. In addition, two parameters rather than one affect the error event lengths: both the information block size k and the encoder memory order m.

3) Bandwidth Efficient Codes

There are two basic ways in which higher rates of information transmission can be achieved over NASA's existing uplink transmission facilities:

i) Higher code rates can be used. This would mean increasing the rate of the inner code from the current 1/2 or 1/3 to a rate closer to unity.

ii) A larger set of modulation symbols can be used. This would mean expanding the current modulation system from QPSK to 8-PSK or 16-PSK modulation.

The most straightforward means of achieving higher rates of information transmission is to increase the rate of the inner code. Recent work by Lin and Kasami has investigated the use of "cascaded codes" which increase the overall system rate from .437 to rates as high as .721 (a maximum 65% increase in information transmission rate). These cascaded codes use a combination of erasure decoding and a decoding failure mechanism to achieve very high reliability. The inner code in a cascaded code is a high rate shortened BCM or Hamming code, while the outer code is an RS code which is capable of errors-and-erasures decoding.

We are currently investigating the use of high rate (2/3 and 3/4) inner convolutional codes along with an outer RS code for this same application. The inner convolutional code can make use of soft decision Viterbi decoding to provide a performance advantage. We are working on a scheme for declaring erasures under certain conditions at the output of the Viterbi decoder, so that an outer errors-and-erasures RS decoder can be used. This scheme must be capable of identifying which portions of the Viterbi decoder output are unreliable. In this way only unreliable

symbols in the outer code are erased. We are also investigating the use of even higher rate (up to rate 8/9) unit-memory inner codes for use with outer RS codes. These codes have an information block size which is matched to the symbol size of the outer code, thus providing a natural coding interface. Schemes for declaring erasures of unreliable information blocks from the inner decoder are being developed for this case as well. Our goal is to achieve even higher rate schemes using inner convolutional codes than those being studied using block codes. We believe a performance advantage will accrue to the convolutional codes due to the availability of soft decision decoding, particularly when unit memory codes are used. Results of this investigation will be included in our next report.

All of the above mentioned schemes maintain the use of QPSK modulation in which one bit of information is transmitted per signal dimension (a QPSK signal is simply two BPSK signals transmitted in phase quadrature). As such, the maximum possible increase in transmission efficiency would be a doubling (100% increase) of the information transmission rate, which could be achieved by using uncoded QPSK modulation instead of an inner code. Any cascaded coding scheme which uses QPSK modulation will achieve something less than a 100% increase in transmission rate. Higher percentage increases in rate can only be achieved by expanding the modulation set to more than four phases.

We have done some preliminary work on combining an inner convolutional code with 8-PSK modulation. A summary of a paper presented at the recent Allerton Conference, along with copies of the transparencies used for the presentation, are included with this report. A rate-2/3 inner convolutional code combined with 8-PSK modulation achieves a transmission

rate of one bit per signal dimension, the same as for uncoded QPSK. However, the coding/modulation combination provides a coding gain over uncoded QPSK modulation, as has been documented in numerous recent articles (see, for example, [1] and [2]). Even higher transmission rates can be achieved. For example, if a rate-5/6 inner convolutional code is combined with two 8-PSK signals, five information bits are sent in four dimensions. This represents a 150% increase in transmission efficiency over the scheme which uses a rate-1/2 inner code. Coding gains (compared with uncoded transmission at the same rate) can also be achieved with the rate-5/6/8-PSK scheme. Expanded modulation sets combined with convolutional coding provide the key to achieving reliable communication at information transmission rates which can even exceed the available channel bandwidth.

In the next phase of our research we will be examining the performance of some of these combined coding/modulation schemes concatenated with an outer RS code to provide even greater reliability at high rates of information transmission. Schemes for erasing certain inner decoder output symbols for errors-and-erasures decoding by the outer decoder will also be investigated.

References

1. G. Ungerboeck, "Channel Coding with Multilevel/Phase Signals", IEEE Trans. Inform. Th., vol. IT-28, pp. 55-67, January 1982.

2. G. D. Forney, Jr., R. G. Gallager, G. R. Lang, F. M. Longstaff, and S. U. Qureshi, "Efficient Modulation for Band-Limited Channels", IEEE Journal on Selected Areas in Comm., vol. SAC-2, pp. 632-647, September 1984.

Appendix

This appendix presents a collection of performance figures for various uplink error control coding systems. In all the figures, the transmission channel is assumed to be AWGN with BPSK modulation.

I. Performance of convolutional codes.

Figure 1 shows simulation results for the bit error rate of convolutional codes of constraint Length $K = (m+1)k = 7$ and of rates $R = 1/2$ and $R = 1/3$ [1]. We also plotted simulation results for the bit error rate of punctured codes of rates $R = 2/3$, $3/4$, and $7/8$ [2,3]. The punctured codes are generated from a constraint Length $K = 7$ rate $R = 1/2$ convolutional code. Three bit quantization is used.

II. Performance of RS codes.

The bit error rate after decoding of RS codes can be estimated by

$$P_b = \frac{d}{N.2} \sum_{i=t+1}^{N} \binom{N}{i} p^i (1-p)^{N-i} , \tag{1}$$

where N and d are the code length and minimum distance of the RS code, $t = \left\lfloor \frac{(d-1)}{2} \right\rfloor$, and p is the channel symbol error rate. Assuming that the symbols of the RS code are over $GF(2^8)$, then $N = 2^8-1 = 255$. For BPSK modulation over an AWGN channel with hard quantization, the channel bit error rate is

$$\varepsilon = Q\left(\sqrt{2R\frac{E_b}{N_o}}\right) , \tag{2}$$

and the channel symbol error rate p in (1) is given by

$$p = 1-(1 - \varepsilon)^8. \tag{3}$$

Using (1) - (3), the bit error rate performance of RS codes with symbols from $GF(2^8)$ is depicted in Figure 2.

III. Performance of concatenated codes for a Viterbi-decoded convolutional inner code and a Reed-Solomon outer code.

We use rate 1/2 and 1/3 constraint length 7 convolutional codes and rate 2/3, 3/4, and 7/8 punctured convolutional codes as inner codes. The inner codes are decoded by the Viterbi decoding algorithm with three bit quantization. The outer code is an RS code with symbols over $GF(2^8)$. The code length is chosen to be $2^8 - 1 = 255$. We denote the overall code rate by R.

To calculate the performance of the concatenated code, we upper bound the outer channel symbol error rate by 8 times the outer channel bit error rate [1]. Then (1) can be used to estimate the bit error rate at the output of the concatenated code.

The code performance for inner code rates of 1/3, 1/2, 2/3, 3/4, and 7/8 and t = 8, 16, and 32 RS codes is plotted in Figures 3, 4, and 5, respectively.

IV. Performance of cascaded codes.

In [4] a cascaded coding scheme is described. The inner code C_1 is a binary (n_1,k_1) code with minimum distance d_1. The outer code is an (n_2,k_2) RS code with symbols from $GF(2^8)$. The performance of the cascaded code is measured by two parameters - the probability of block erasure, P_{es}, and the probability of block decoding error, P_{er}.

If the cascaded code is used in an ARQ system, then the erased block can be retransmitted. In this case the bit error rate at the output of the cascaded code decoder is given by

$$P_b \doteq \frac{d_2}{2n_2} P_{er}, \qquad (4)$$

and the system throughout is given by (for selective-repeat ARQ)

$$\eta \doteq \frac{k_1}{n_1}\frac{k_2}{n_2}(1-P_{es}) = R(1-P_{es}). \qquad (5)$$

On the other hand, if the cascaded code is used in an FEC system, the bit error rate after decoding is

$$P_b \doteq \frac{d_2}{2n_2}(P_{er} + P_{es}). \qquad (6)$$

In Table 1, we list the inner codes used in the cascaded code.

Table 1. Inner Codes

	n_1	k_1	d_1	generator polynomial
$C_1(1)$ Shortened BCH	59	40	8	$(1+x)(1+x+x^6)$ $(1+x+x^2+x^4+x^6)$ $(1+x+x^2+x^5+x^6)$
$C_1(2)$ Shortened BCH	53	40	6	$(1+x)(1+x+x^6)$ $(1+x+x^2+x^4+x^6)$
$C_1(3)$ Shortened BCH	61	48	6	$(1+x)(1+x+x^6)$ $(1+x+x^2+x^4+x^6)$
$C_1(4)$ Shortened Hamming	30	24	4	$(1+x)(1+x+x^6)$

Figures 6-9 show the performance of the cascaded codes when used in a selective-repeat ARQ system.

Figures 10-13 show the performance of the cascaded code when used in an FEC system.

V. Conclusions.

When a cascaded code is used in ARQ systems, it provides extremely low bit error rates, but it is only useful for large E_b/N_o (larger than 5 dB). If $E_b/N_o < 5$ dB, the system throughput becomes very small.

If a cascaded code is used in an FEC system, high system reliability (low bit error rate) cannot be achieved due to erasures in the outer code.

Comparing the results in sections III and IV, we see that the concatenated code with a Viterbi decoded inner convolutional code and an RS outer code performs somewhat better than the cascade code.

References

[1] Odenwalder, J. P., "*Error Control Coding Handbook*", Final Report under Contract No. F44620-76-C-0056 for USAF, LINKABIT Corp., July 1976.

[2] Paaske, E., "Short binary convolutional codes with maximal free distance for rates 2/3 and 3/4", *IEEE Trans. on Inform. Theory*, vol. IT-20, pp. 683-689, Sept. 1974.

[3] Yasuda, Y., Kashiki, K., and Hirata, Y., "High-rate punctured convolutional codes for soft decision Viterbi decoding", *IEEE Trans. on Commun.*, vol. COM-32, pp. 315-319, March 1984.

[4] Lin, S., and Kasami, T., "A cascaded coding scheme for error control", *NASA Technical Report*, October 1985.

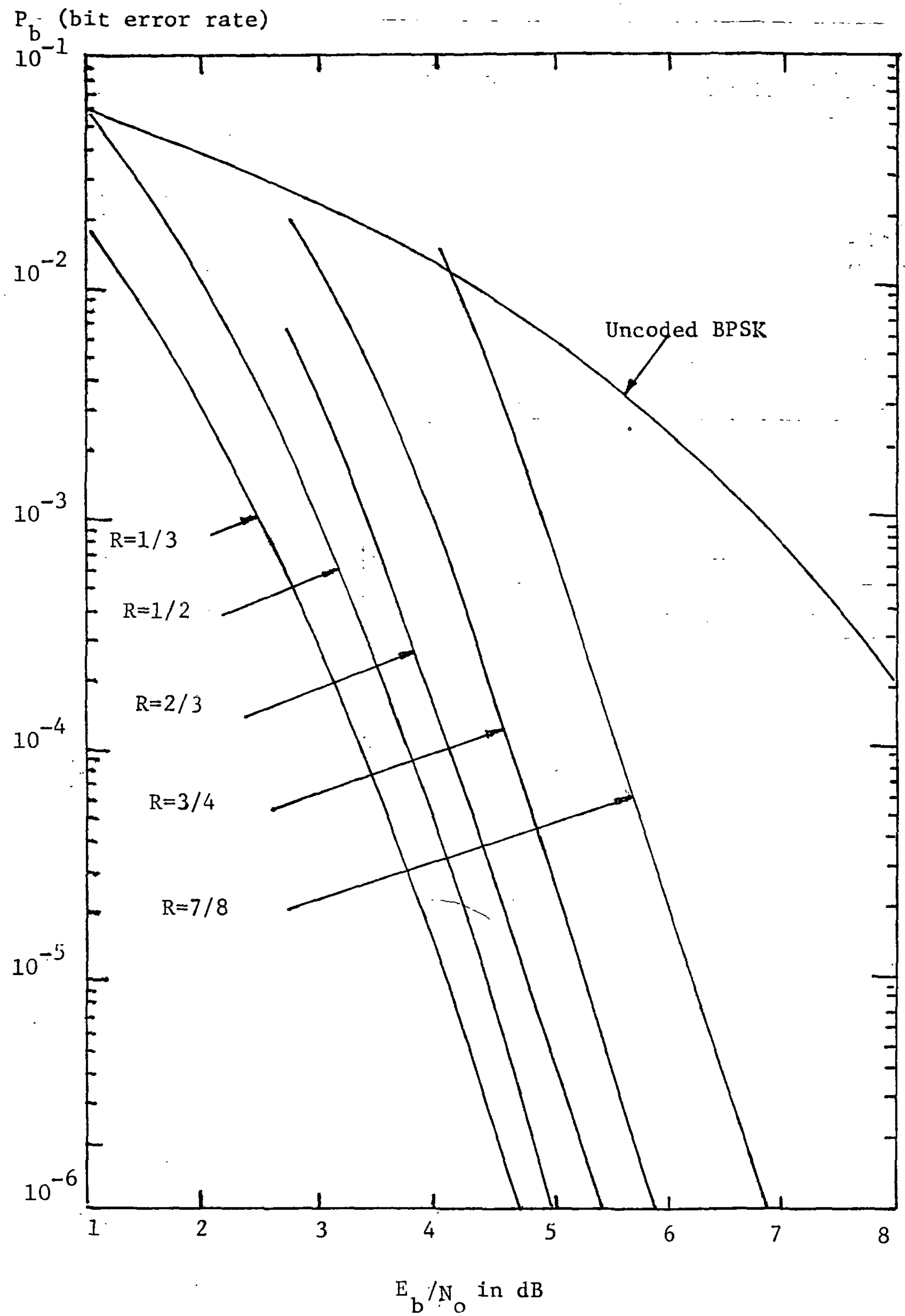

Figure 1. Performance of convolutional codes of constraint length K=7. Three bit quantization is used.

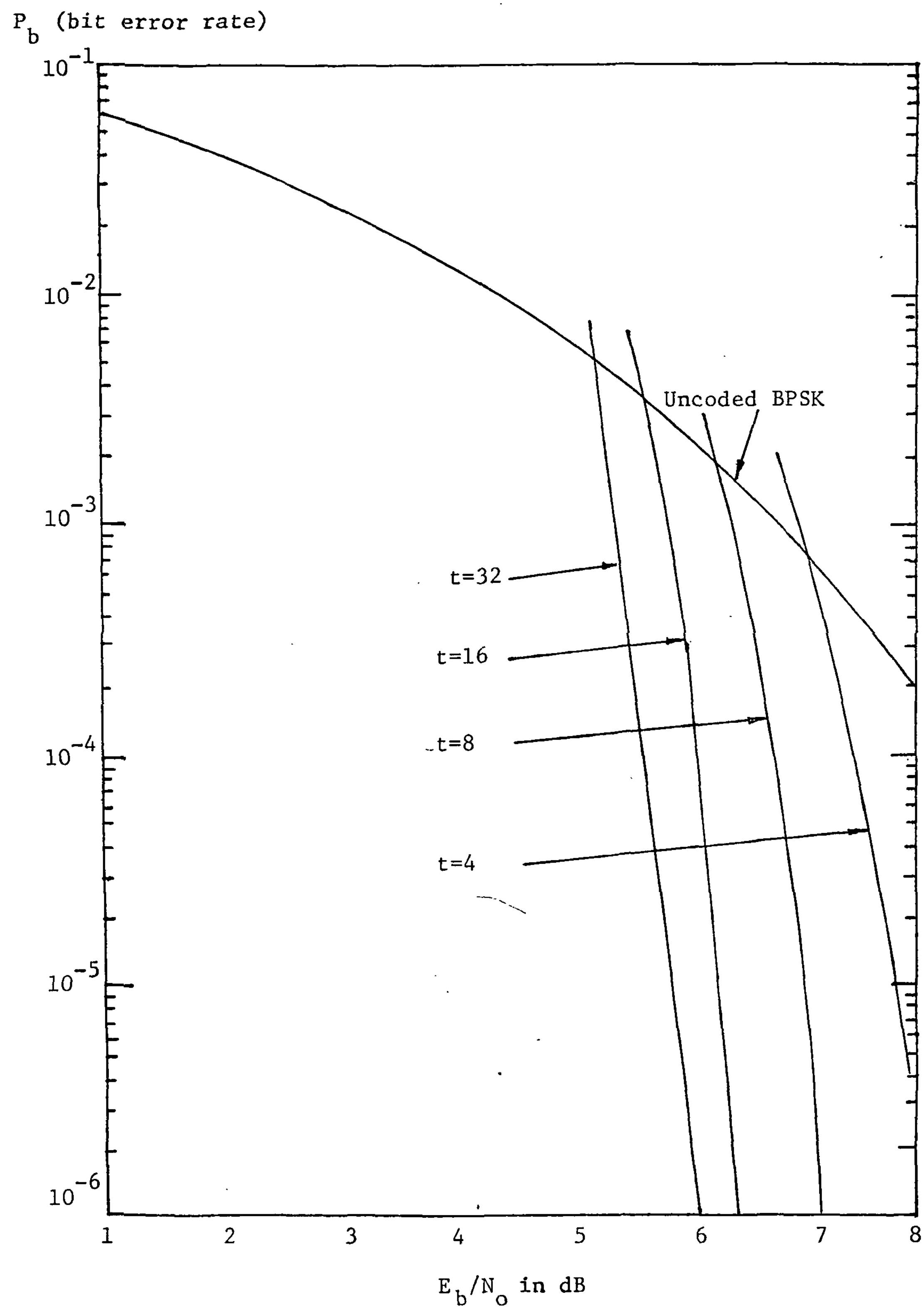

Figure 2. Performance of RS codes with symbols from $GF(2^8)$.

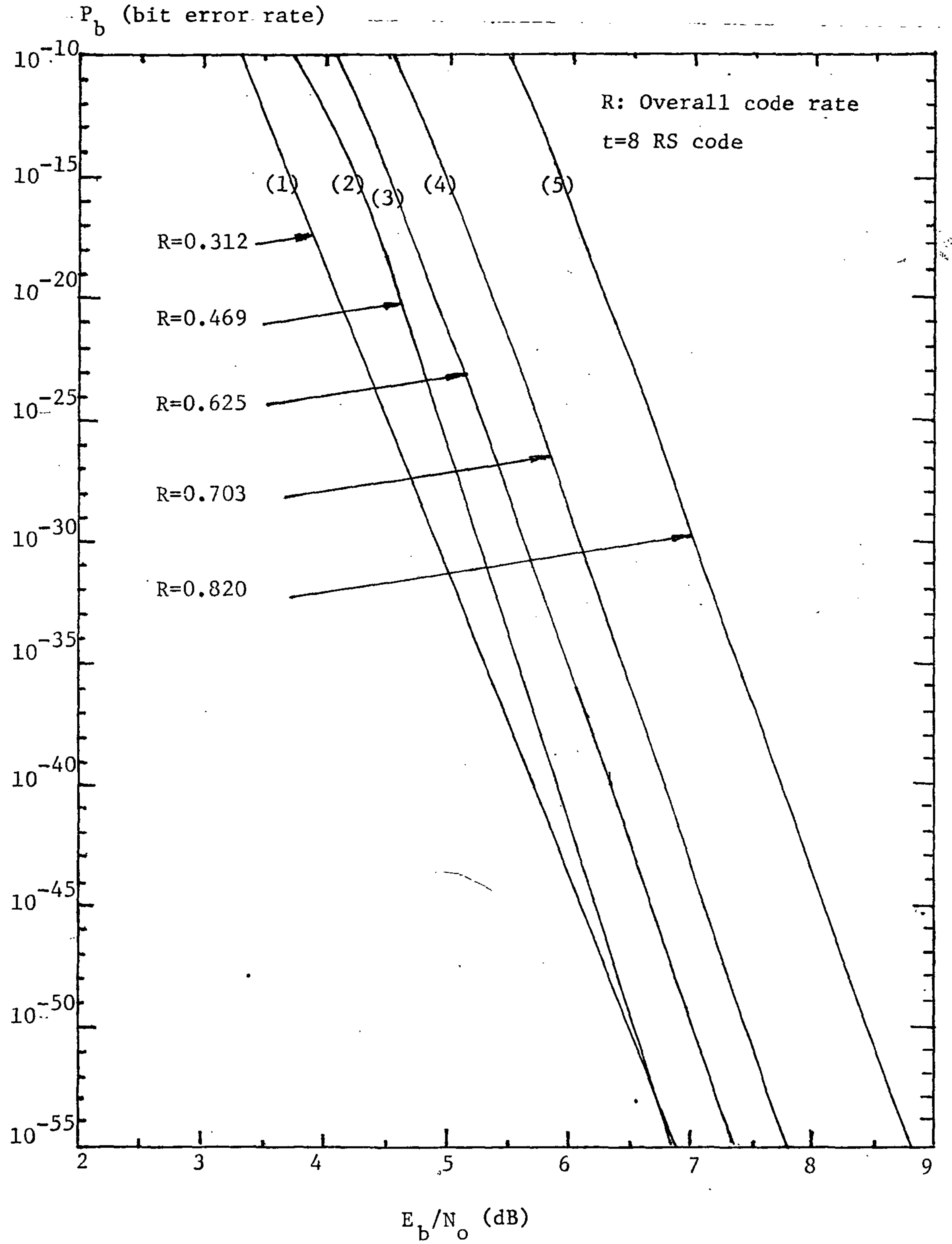

Figure 3. Performance of concatenated code with (255,239) RS outer code and Viterbi decoded inner convolutional code of constraint length 7 and of rate: (1) R_1=1/3; (2) R_1=1/2; (3) R_1=2/3; (4) R_1=3/4; (5) R_1=7/8.

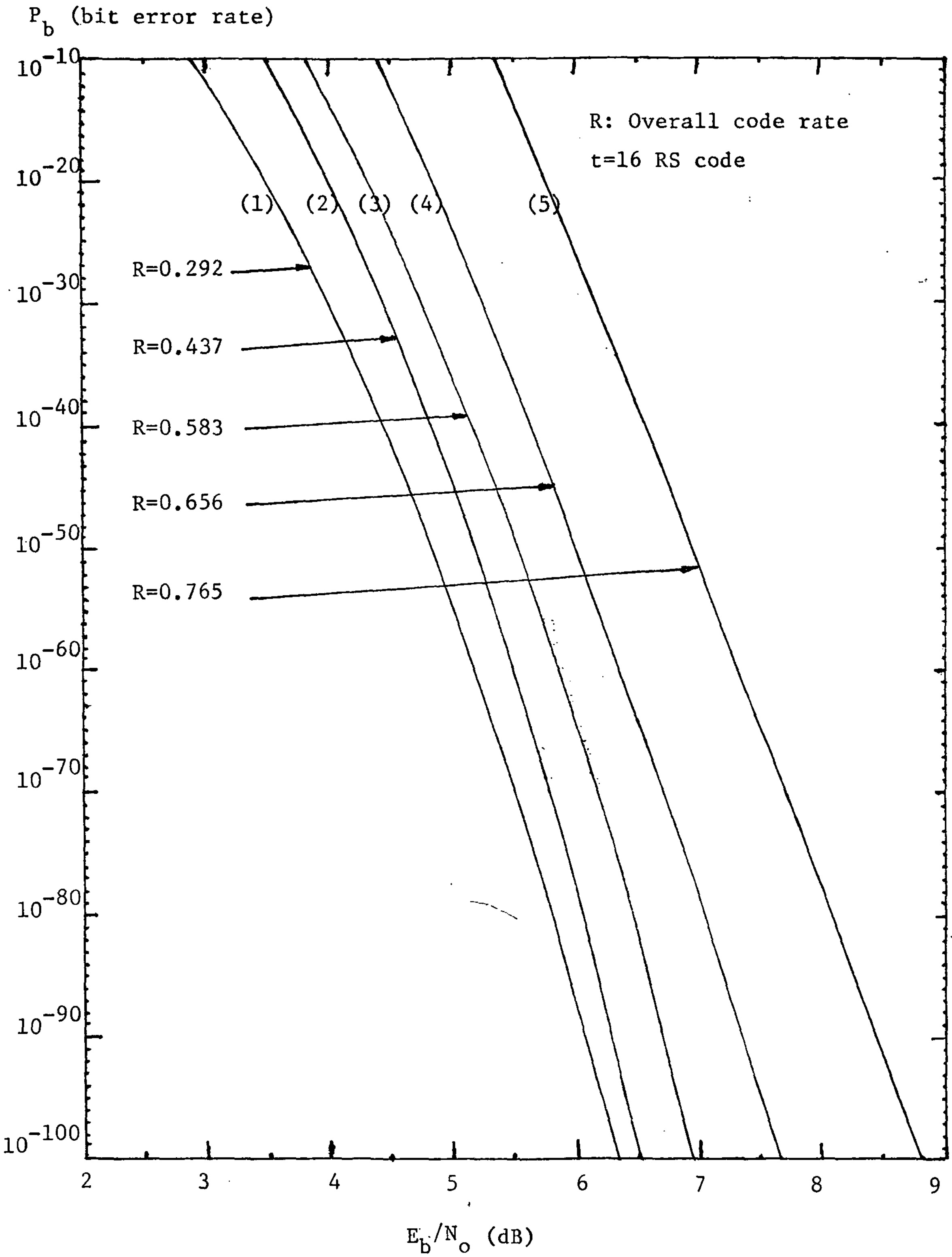

Figure 4. Performance of concatenated code with (255,223) RS outer code and Viterbi decoded inner convolutional code of constraint length 7 and of rate: (1) $R_1=1/3$; (2) $R_1=1/2$; (3) $R_1=2/3$; (4) 3/4; (5) 7/8.

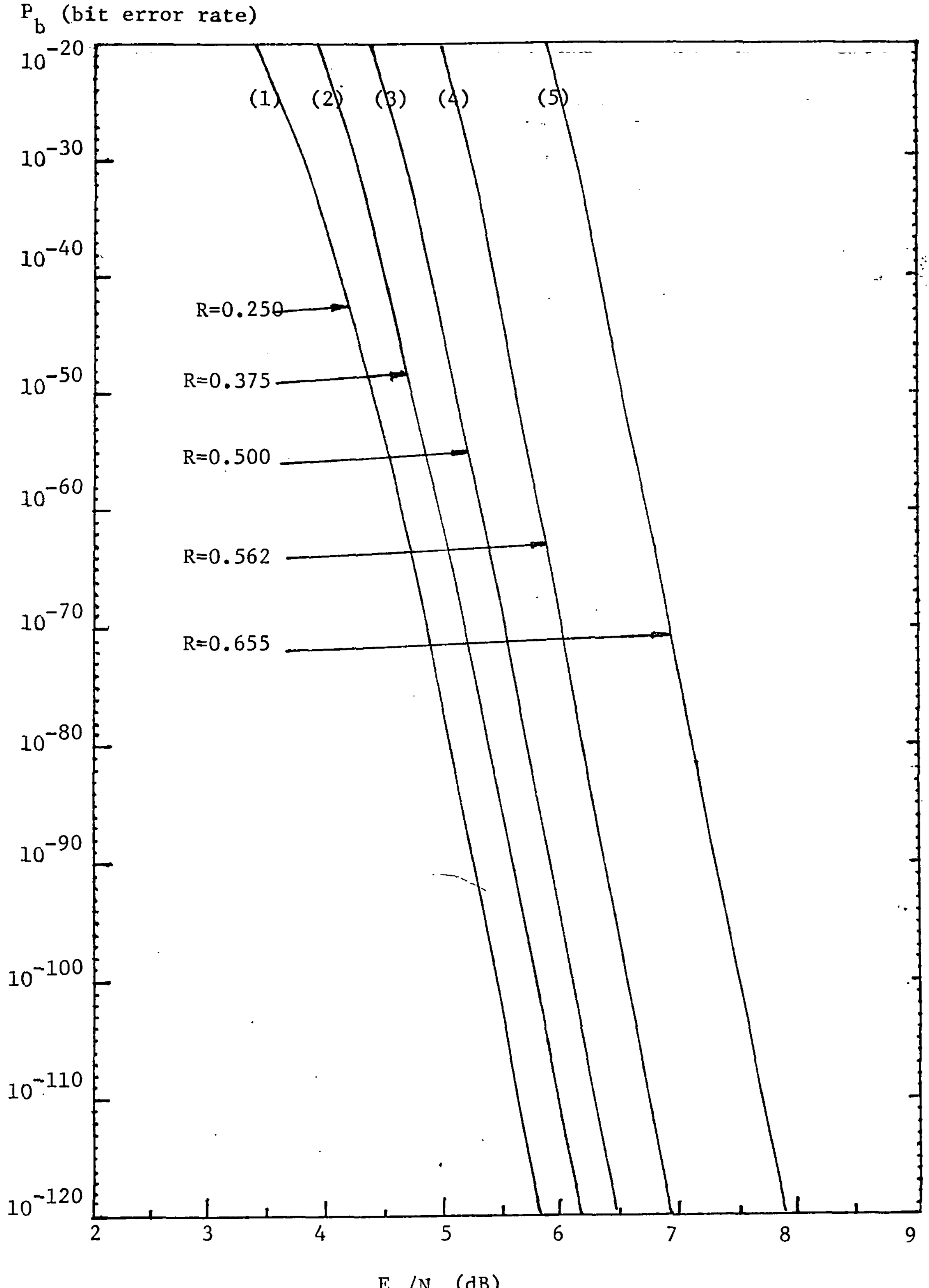

Figure 5. Performance of concatenated code with (255,191) RS outer code and Viterbi decoded inner convolutional code of constraint length 7 and of rate: (1) R_1=1/3; (2) R_1=1/2; (3) R_1=2/3; (4) R_1=3/4; (5) R_1=7/8.

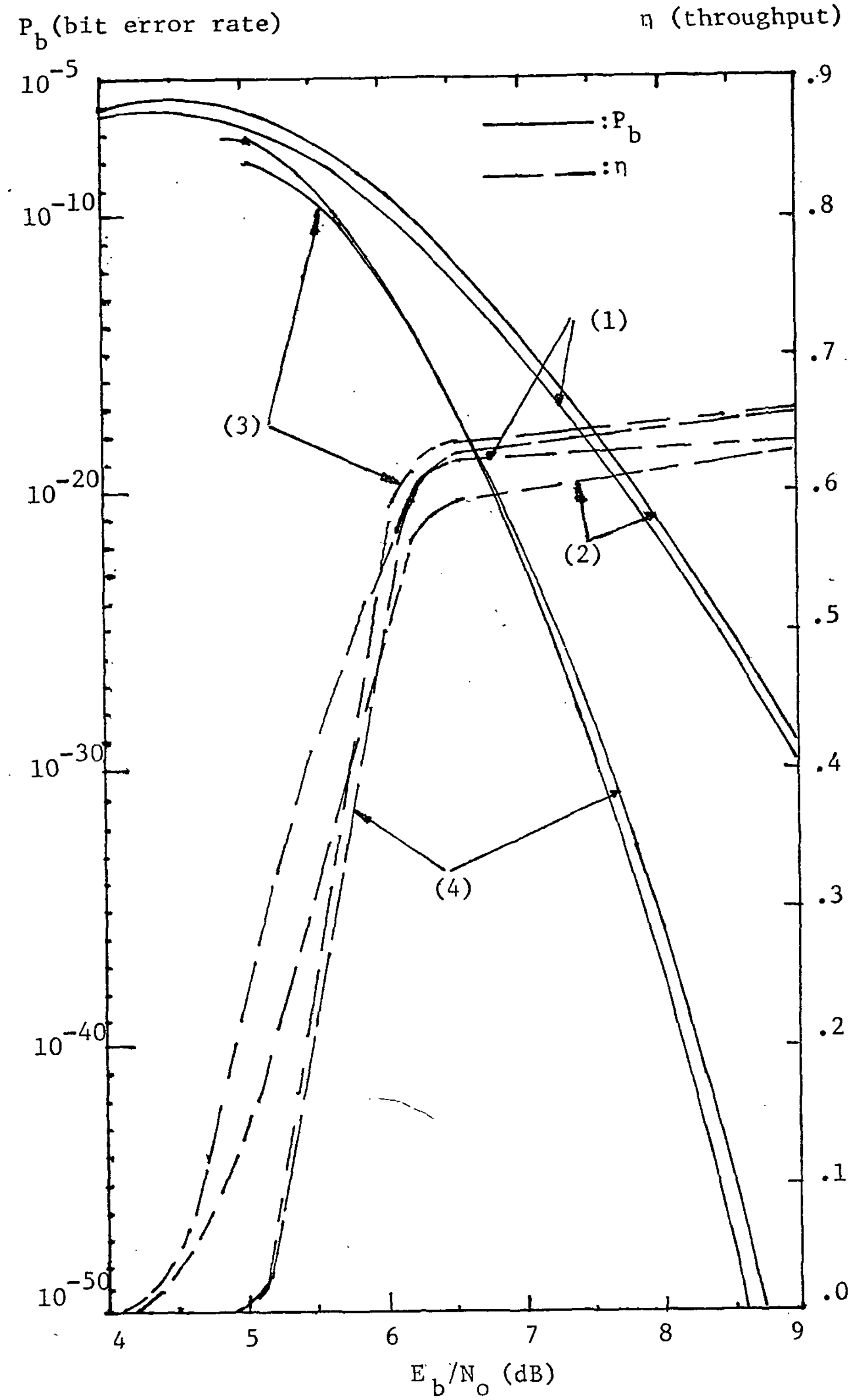

Figure 6. Performance of cascaded codes (ARQ). (1) Inner code: $C_1(1)$, outer code: (255,238) RS code, E, Id=1, R=0.633; (2) Inner code: $C_1(1)$, outer code: (255,236) RS code, L, Id=1, R=0.627; (3) Inner code: $C_1(1)$, outer code: (255,247) RS code, E, Id=5, R=0.657; (4) Inner code: $C_1(1)$, outer code: (255,246) RS code, L, Id=5, R=0.654.

Note: E:Erasure-only inner decoding; L:LIA-inner decoding; Id: Degree of interleaving; R:overall code rate.

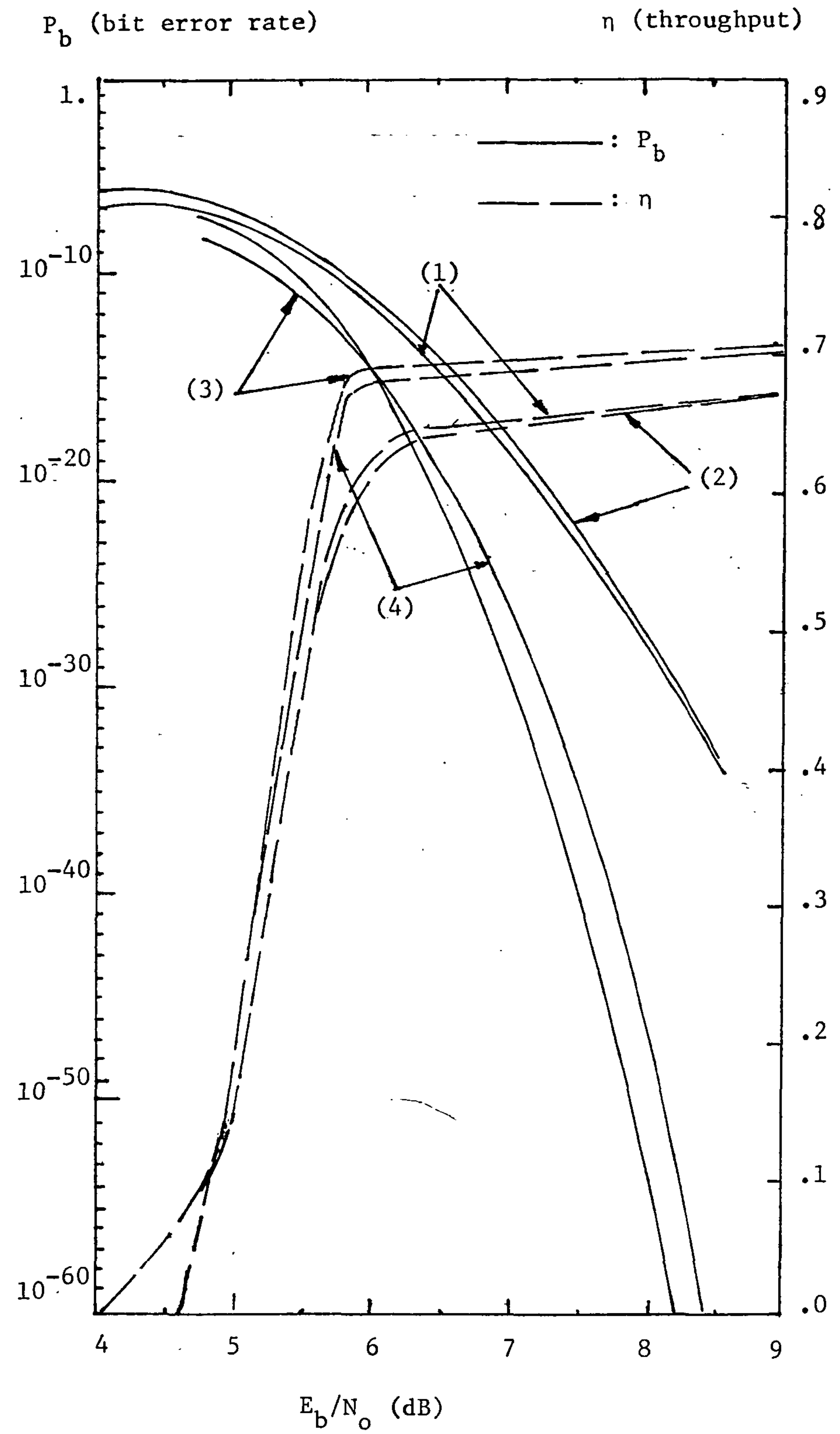

Figure 7. Performance of cascaded codes (ARQ). (1) Inner code: $C_1(2)$, Outer code: (255,228) RS code, E, Id=1, R=0.675; (2) Inner code: $C_1(2)$, Outer code: (255,228) RS code, L, Id=1, R=0.675; (3) Inner code: $C_1(2)$, Outer code: (255,239) RS code, E, Id=5, R=0.707; (4) Inner code: $C_1(2)$, Outer code: (255,237) RS code, L, Id=5, R=0.701.

Note: E: Erasure-only inner decoding; L:LIA-inner decoding; Id: Degree of interleaving; R: overall rate.

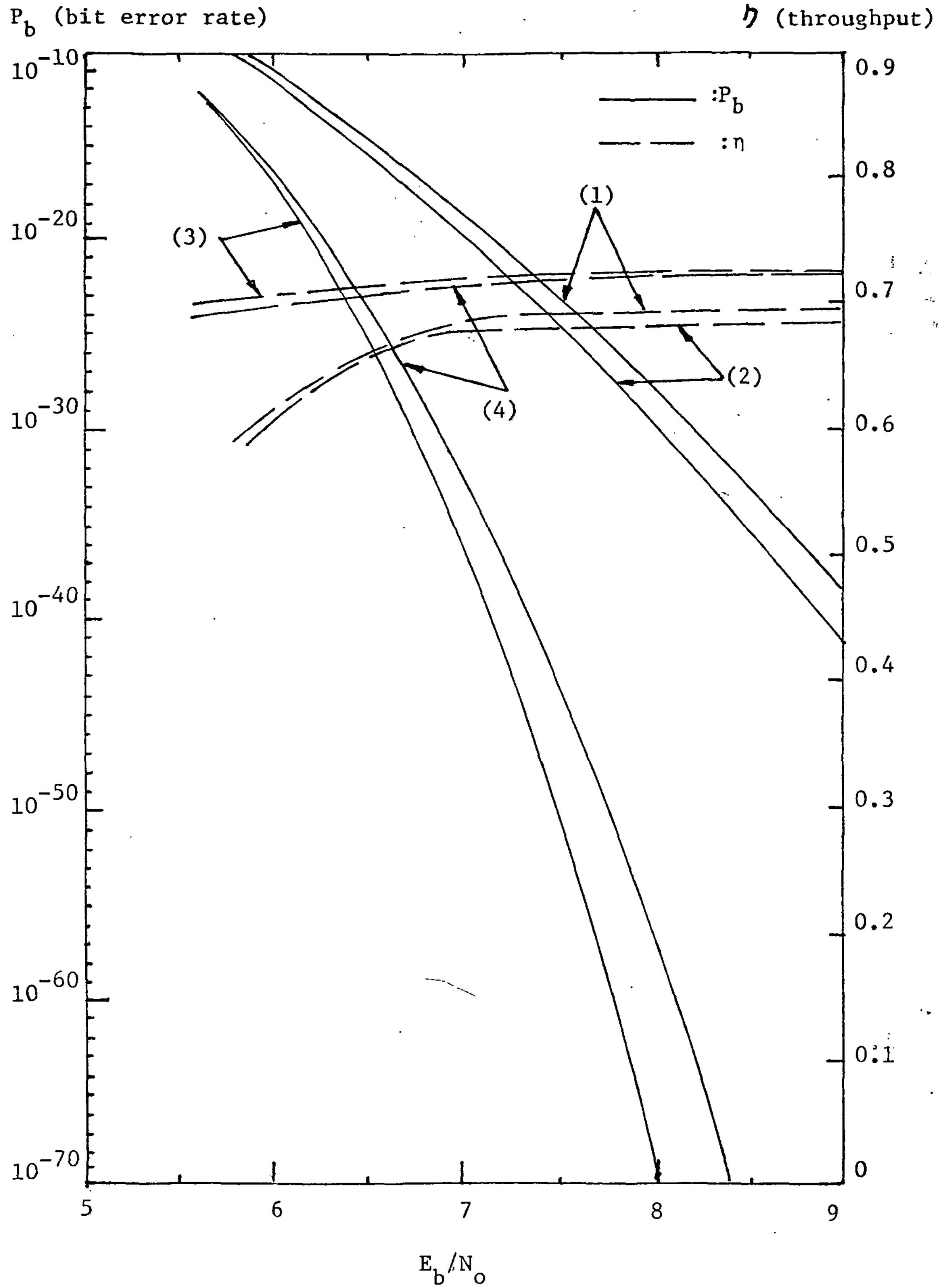

Figure 8. Performance of cascaded codes (ARQ). (1) Inner code: $C_1(3)$, outer code: (255,220) RS code, E, Id=1, R=0.678; (2) Inner code: $C_1(3)$, outer code: (255,221) RS code, L, Id=1, R=0.688, (3) Inner code: $C_1(3)$, outer code: (255,234) RS code, E, Id=6, R=0.721; (4) Inner code: $C_1(3)$, outer code: (255,233) RS code, L, Id=6, R=0.718.

Note: E: Erasure-only inner decoding; L:LIA inner decoding; Id: Degree of interleaving; R: overall code rate.

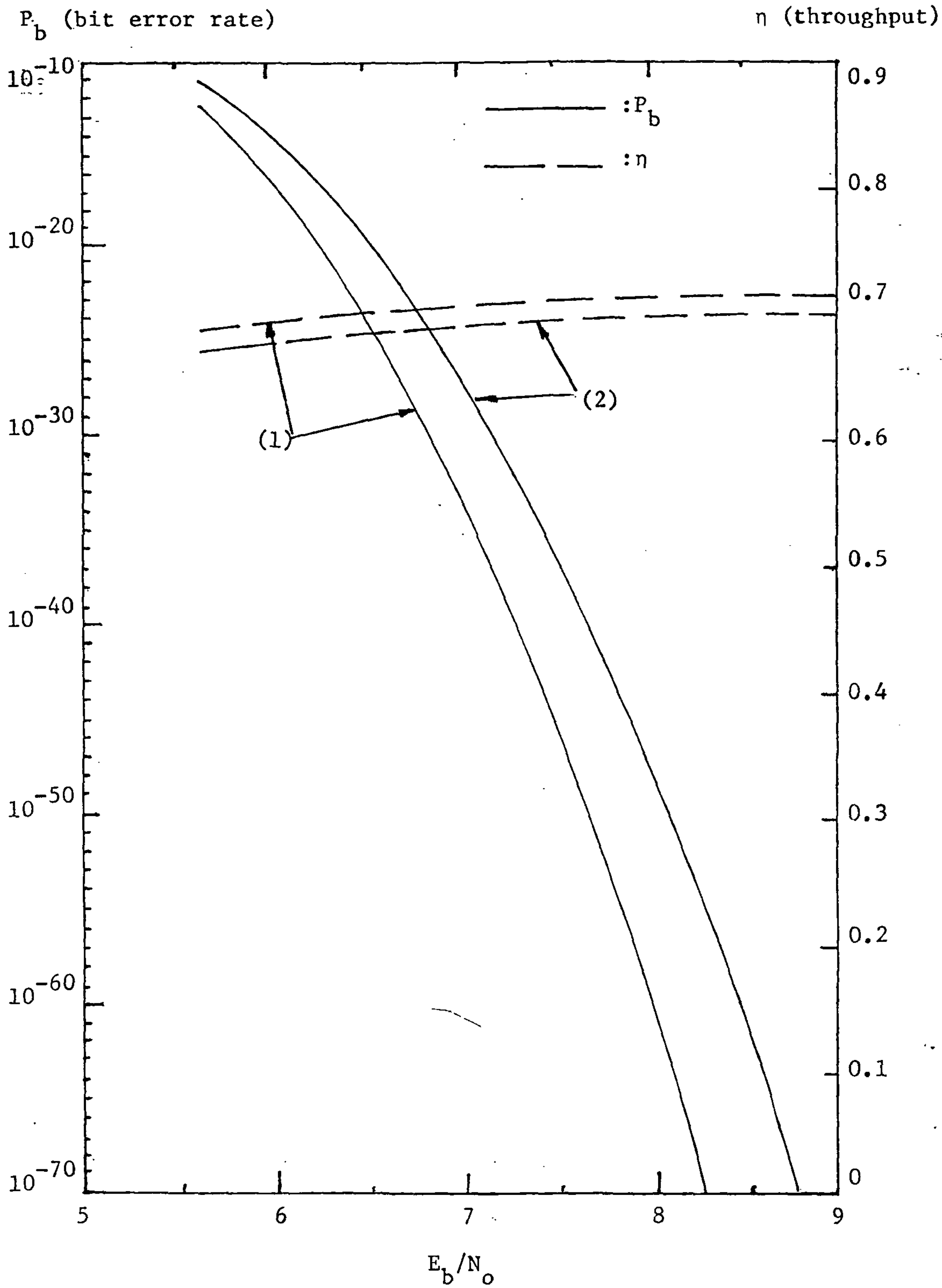

Figure 9. Performance of cascaded code (ARQ).
(1) Inner code: $C_1(4)$, outer code: (255,228) RS code, E, Id=3, R=0.715; (2) Inner code: $C_1(4)$, outer code: (255,225) RS code, L, Id=3, R=0.706.

Note: E: Erasure-only inner decoding; L:LIA inner decoding; Id: Degree of interleaving; R: overall code rate.

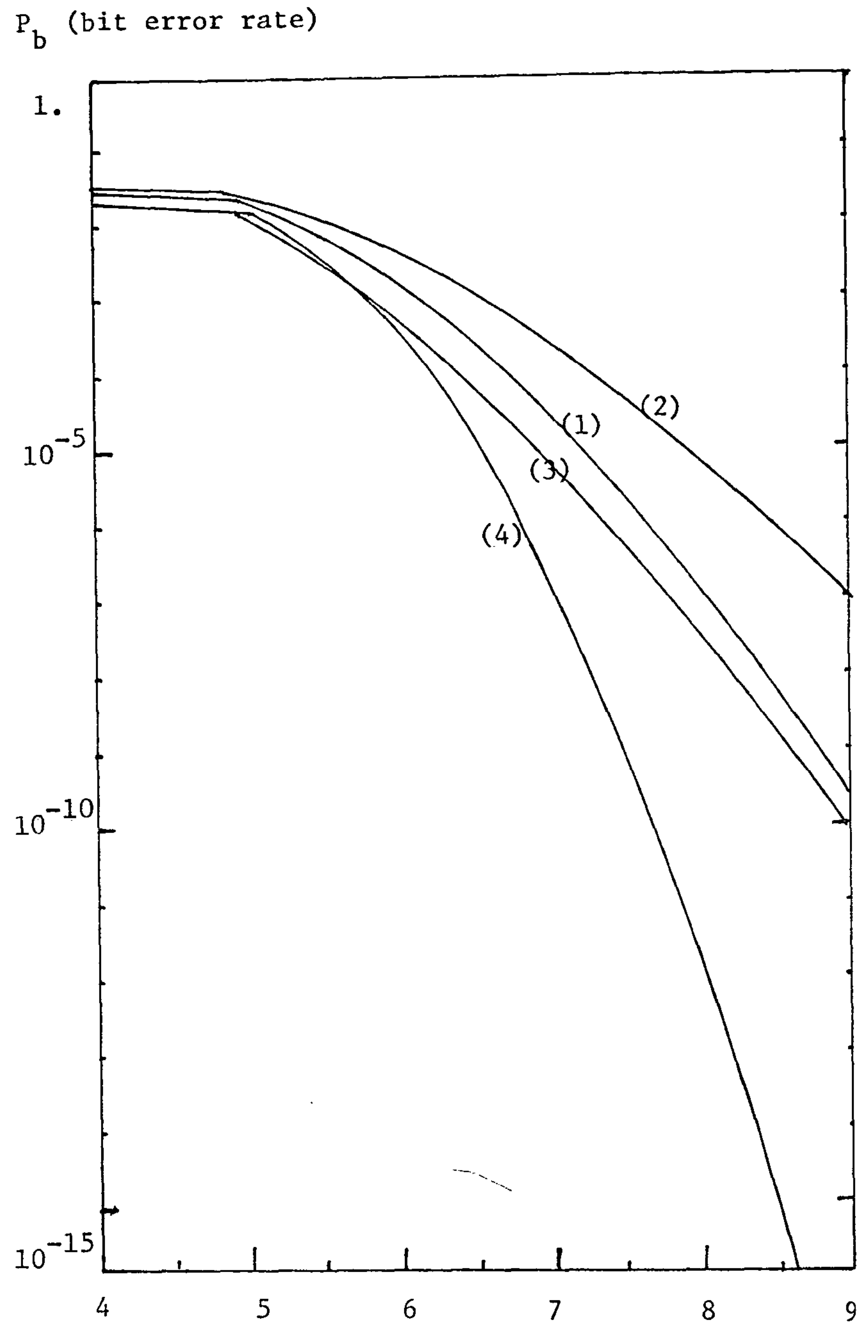

Figure 10. Performance of cascoded code (FEC). (1) Inner code: $C_1(1)$, outer code: (255,238) RS code, E, Id=1, R=0.633; (2) Inner code: $C_1(1)$, outer code: (255,236) RS code, L, Id=1, R=0.627; (3) Inner code: $C_1(1)$, outer code: (255,247) RS code, E, Id=5, R=0.657; (4) Inner code: $C_1(1)$, outer code: (255,246) RS code, L, Id=5, R=0.654.

Note: E:Erasure-only inner decoding; L:LIA inner decoding;
Id: Degree of interleaving; R: overall code rate.

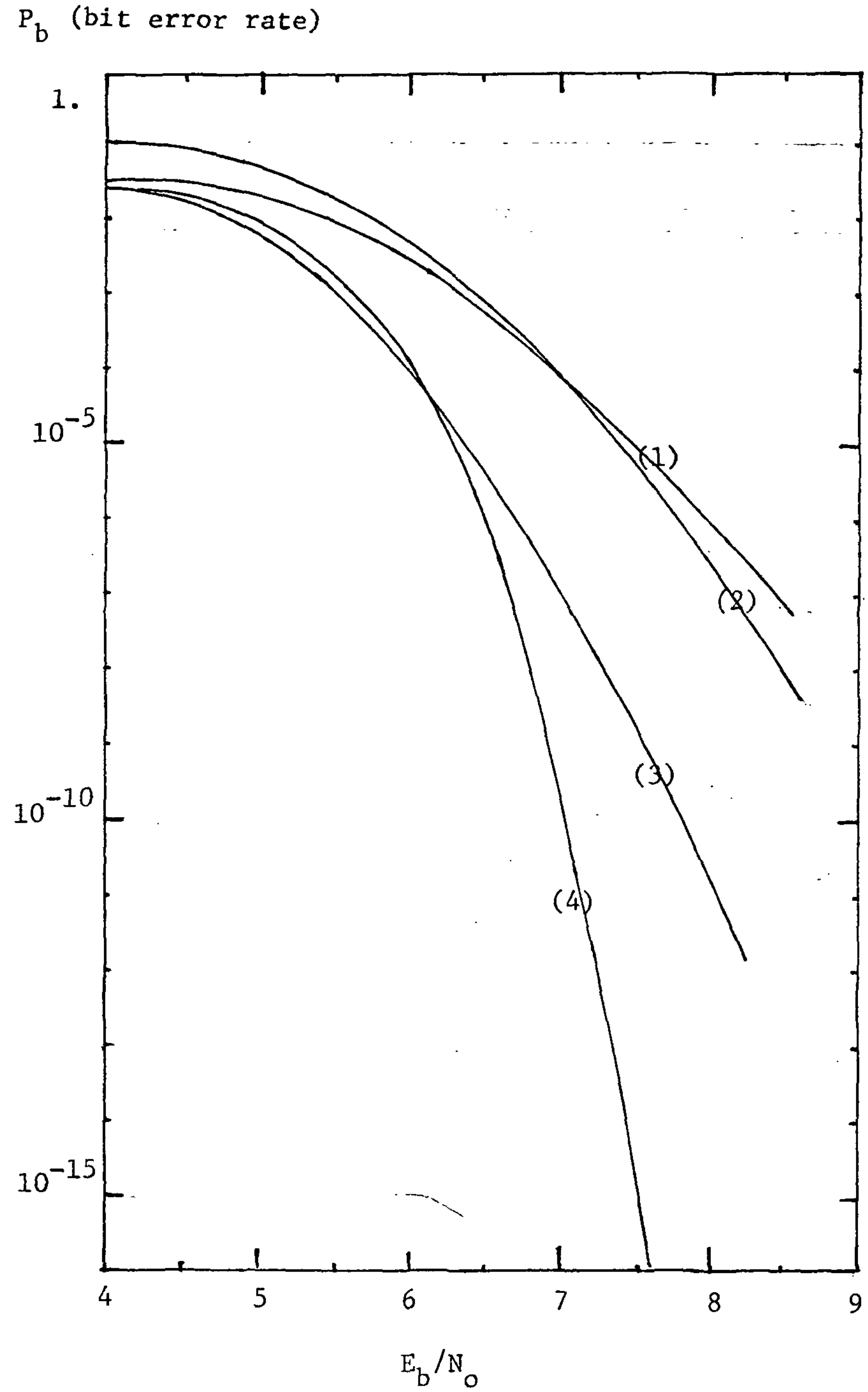

Figure 11. Performance of cascaded code (FEC). (1) Inner code: $C_1(2)$, outer code: (255,228) RS code, E, Id=1, R=0.675; (2) Inner code: $C_1^1(2)$, outer code: (255,228) RS code, L, Id=1, R=0.675; (3) Inner code: $C_1^1(2)$, outer code: (255,239) RS code, E, Id=5, R=0.707; (4) Inner code: $C_1^1(2)$, outer code: (255,237) RS code, L, Id=5, R=0.701.

Note: E: Erasure-only inner decoding; L:LIA inner decoding;
Id: Degree of interleaving; R: overall code rate.

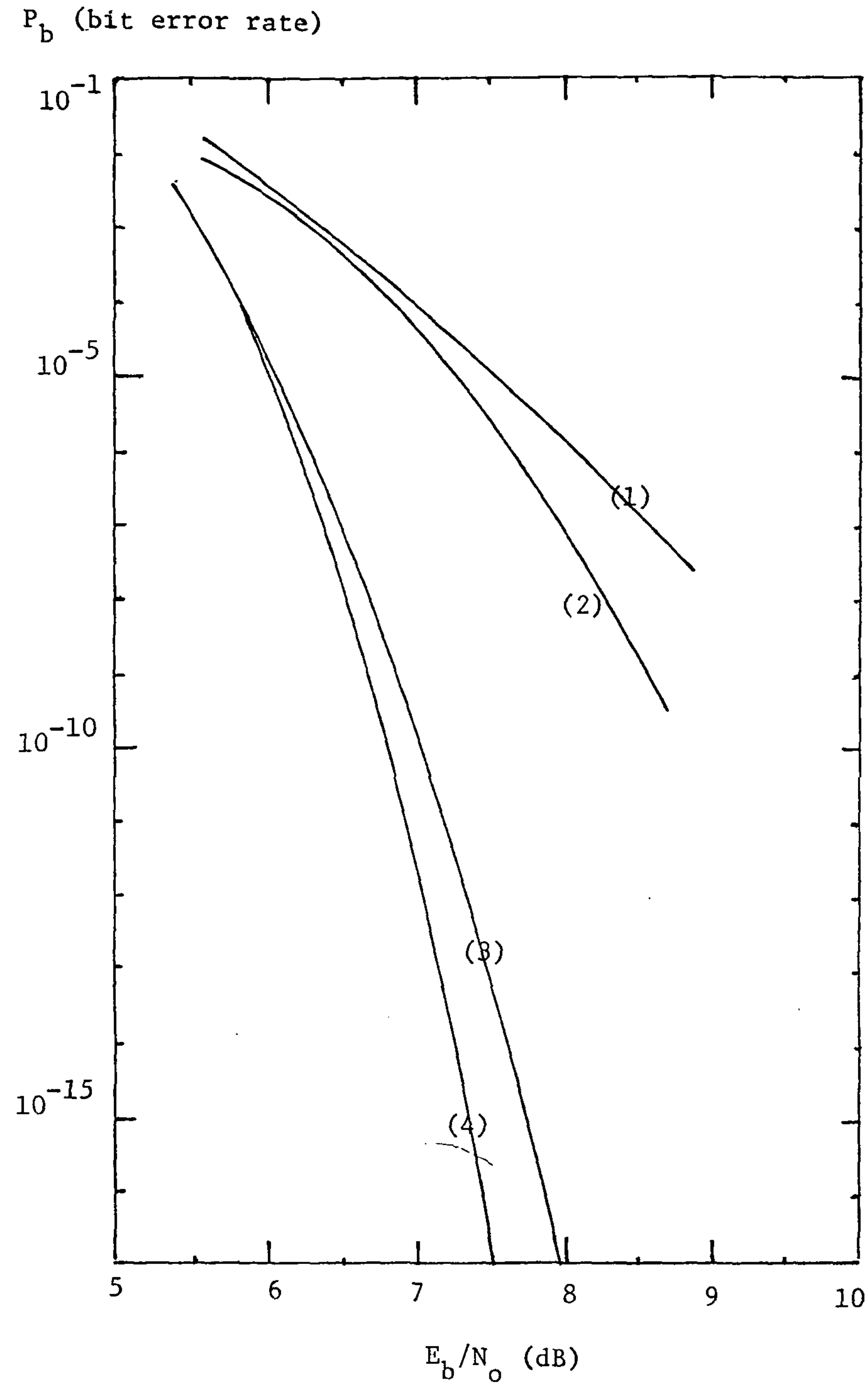

Figure 12. Performance of cascaded code (FEC). (1) Inner code: $C_1(3)$, outer code: (255,220) RS code, E, Id=1, R=0.678; (2) Inner code: $C_1(3)$, outer code: (255,221) RS code, L, Id=1, R=0.688; (3) Inner code: $C_1(3)$, outer code: (255,234) RS code, E, Id=6, R=0.721; (4) Inner code: $C_1(3)$, outer code: (255,233) RS code, L, Id=6, R=0.718.

Note: E: Erasure-only inner decoding; L:LIA inner decoding;
ID: Degree of interleaving; R: overall code rate.

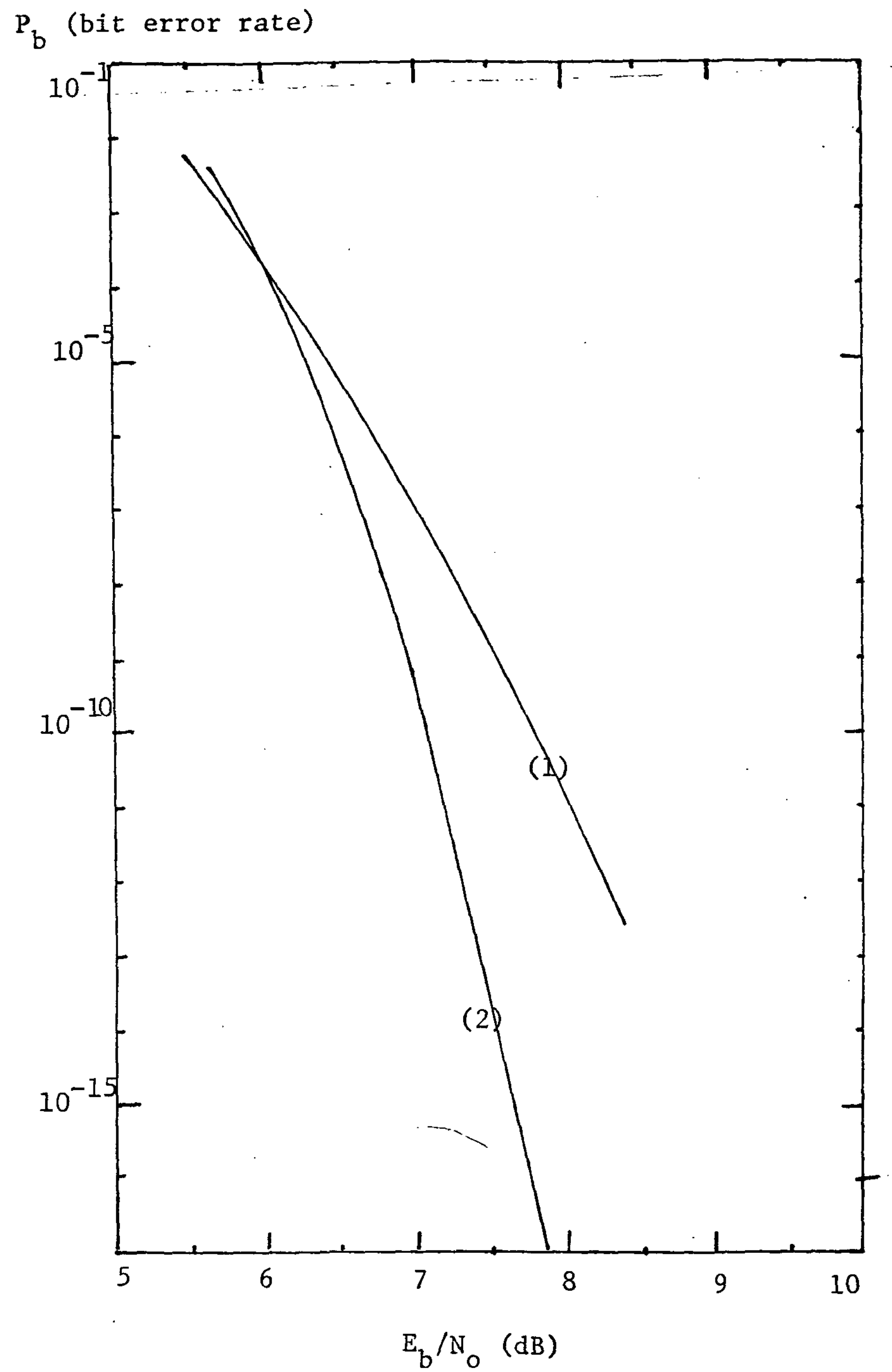

Figure 13. Performance of cascaded code (FEC). (1) Inner code: $C_1(4)$, outer code: (255,228) RS code, E, Id=3, R=0.715; (2) Inner code: $C_1(4)$, outer code: (255,225) RS code, L, Id=3, R=0.706.

Note: E: Erasure-only inner decoding; L:LIA inner decoding; Id: Degree of interleaving; R: overall code rate.